ME 235A Finite Element Analysis
Fall, 2001

Solution to Problem Set #1
Oct. 10 2001

1. Use the definitions of $a(\cdot,\cdot)$ and $(\cdot,\cdot)$ to verify the properties of symmetry and bilinearity.

symmetry:

$$a(u,v)=\int_0^1 u_{,x}v_{,x}dx=\int_0^1 u_{,x}v_{,x}dx=a(u,v)$$

$$(u,v)=\int_0^1 uvdx=\int_0^1 vudx=(v,u)$$

bilinearity:

$$a(c_1u+c_2v,w)=\int_0^1 (c_1u+c_2v)_{,x}w_{,x}dx$$

$$=c_1\int_0^1 u_{,x}w_{,x}dx+c_2\int_0^1 v_{,x}w_{,x}dx=c_1a(u,w)+c_2a(v,w)$$

$$a(u,c_1v+c_2w)=a(c_1v+c_2w,u)=c_1a(u,v)+c_2a(u,w)$$

$$(c_1u+c_2v,w)=\int_0^1 (c_1u+c_2v)wdx$$

$$=c_1\int_0^1 uwdx+c_2\int_0^1 vwdx=c_1(u,w)+c_2(v,w)$$

$$(u,c_1v+c_2w)=(c_1v+c_2w,u)=c_1(u,v)+c_2(u,w)$$

where c_1 and c_2 are constants.

2. (The problem is not restated)

$$\int_0^1 w_{,x}u_{,x}dx=\int_0^1 wfdx+w(0)h$$

$$0=-\sum_{A=1}^{n}\int_{x_A}^{x_{A+1}} w_{,x}u_{,x}dx+\sum_{A=1}^{n}\int_0^1 wfdx+w(0)h$$

$$=\sum_{A=1}^{n}\left(\int_{x_A}^{x_{A+1}} wu_{,xx}dx-wu_{,x}\Big|_{x_A^+}^{x_{A+1}^-}\right)+\sum_{A=1}^{n}\int_{x_A}^{x_{A+1}} wfdx+w(0)h$$

$$=\sum_{A=1}^{n}\int_{x_A}^{x_{A+1}} w(u_{,xx}+f)dx+w(0)h+\sum_{A=1}^{n}[w(x_A)u_{,x}(x_A^+)-w(x_{A+1})u_{,x}(x_{A+1}^-)]+w(0)h$$

$$0=\sum_{A=1}^{n}\int_{x_A}^{x_{A+1}} w(u_{,xx}+f)dx+w(0)[u_{,x}(0^+)+h]+\sum_{A=2}^{n}w(x_A)[u_{,x}(x_A^+)-u_{,x}(x_A^-)]$$

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3. Consider the BVP:

$$-(p(x)\, u_{,x})_{,x} + q(x)\, u = f \qquad x \in [0,1] \qquad (1)$$
$$-p(0)\, u_{,x}(0) - \alpha\, u(0) = 0 \qquad (2)$$
$$p(1)\, u_{,x}(1) + \beta\, u(1) = 0 \qquad (3)$$

where p, q, and f are given functions and α and β are constants, and $p(0)$ and $p(1)$ are nonzero.

1. Define the spaces S and V
2. Obtain the variational equation
3. State the weak form (W) of the problem

* Note that the boundary conditions (2) and (3) are both Neumann.

1. $S = V = \{u \mid u \in H^1\}$

2. $0 = \int_0^1 w[(-(p(x)u_{,x})_{,x} + q(x)u - f]dx$

$$\int_0^1 w_{,x} p(x) u_{,x} dx + \int_0^1 w q(x) u dx = \int_0^1 w f dx - w(0) p(0) u_{,x}(0) + w(1) p(1) u_{,x}(1)$$

$$\int_0^1 w_{,x} p(x) u_{,x} dx + \int_0^1 w q(x) u dx = \int_0^1 w f dx + w(0)\alpha u(0) - w(1)\beta u(1) \qquad (*)$$

3. (W) Given the functions p, q, and f, and the constants α and β, find $u \in S$ such that the variational equation (*) is satisfied for $\forall\, w \in V$.